

Novel scaling laws to derive spatially resolved flare and CME parameters from sun-as-a-star observables

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ABSTRACT

Coronal mass ejections (CMEs) are often associated with X-ray (SXR) flares powered by magnetic reconnection in the low-corona, while the CME shocks in the upper corona and interplanetary (IP) space accelerate electrons often producing the type-II radio bursts. The CME and the reconnection event are part of the same energy release process as highlighted by the correlation between reconnection flux (ϕ_{rec}) that quantifies the strength of the released magnetic free energy during SXR flare, and the CME kinetic energy that drives the IP shocks leading to type-II bursts. Unlike the sun, these physical parameters cannot be directly inferred in stellar observations. Hence, scaling laws between unresolved sun-as-a-star observables, namely SXR luminosity (L_X) and type-II luminosity (L_R), and the physical properties of the associated dynamical events are crucial. Such scaling laws also provide insights into the interconnections between the particle acceleration processes across low-corona to IP space during solar-stellar ‘flare- CME- type-II’ events. Using long-term solar data in SXR to radio waveband, we derive a scaling law between two novel power metrics for the flare and CME-associated processes. The metrics of ‘flare power’ ($P_{\text{flare}} = \sqrt{L_X \phi_{\text{rec}}}$) and ‘CME power’ ($P_{\text{CME}} = \sqrt{L_R V_{\text{CME}}^2}$), where V_{CME} is the CME speed, scale as $P_{\text{flare}} \propto P_{\text{CME}}^{0.76 \pm 0.04}$. Besides, L_X and ϕ_{rec} show power-law trends with P_{CME} with indices of 1.12 ± 0.05 and 0.61 ± 0.05 respectively. These power-laws help infer the spatially resolved physical parameters, V_{CME} and ϕ_{rec} , from disk-averaged observables, L_X and L_R during solar-stellar ‘flare- CME- type-II’ events.

Key words. stars: flare - stars: activity - stars: magnetic field - stars: coronae - radio continuum: stars - stars: low-mass

1. Introduction

Coronal mass ejections (CMEs) are a major driver of space weather events in solar and stellar environments (see, Howard et al. 2023, for an overview). A CME can produce shocks across corona out to interplanetary (IP) space, accelerating electrons which often produce meter-kilometer (m-km) band type-II bursts (Wild & McCready 1950; Wild 1970; McLean & Labrum 1985; Gopalswamy 2011; Miteva et al. 2017; Alvarado-Gómez et al. 2022). Depending on the heights of shock formation, the type-II bursts may have a start and end frequency anywhere within the m-km range. The CMEs associated with the type-II bursts are generally much faster and wider than an average CME, with 78% of them associated with major solar energetic particle (SEP) events (Gopalswamy et al. 2005; Kumari et al. 2023). The type-II associated CMEs are also accompanied by relatively intense soft X-ray (SXR) flares (Gopalswamy 2006). Hence, the type-II bursts are highly sought after in active stars as a CME shock signature (e.g. Osten & Bastian 2008; Crosley & Osten 2018; Villadsen et al. 2014).

When the type-II burst conveys the strength of particle acceleration caused by the CME shocks, the associated SXR flare is produced by the heating of the post-reconnection magnetic field structures in the low-corona. Hence the peak SXR luminosity (L_X) correlates well with the reconnection flux (ϕ_{rec}), which is a proxy to the strength of the reconnection-driven currents that drive the local heating (Kazachenko et al. 2017; Sindhuja & Gopalswamy 2020). Gopalswamy et al. (2018a) showed that

ϕ_{rec} correlates well with CME kinetic energy, revealing a link between the low-coronal and IP space impacts of the energy release event, of which the CME and the flare are part of (hereafter, ‘flare- CME- type-II’ event). However, there exist no scaling laws involving L_X , type-II luminosity (L_R), and the physical properties of the flare and the CME, that could reveal any links between the mechanisms and drivers of particle acceleration across corona to IP space. Besides, since the flare and type-II burst outshine the quiet sun during a ‘flare- CME- type-II’ event, L_X and L_R are sun-as-a-star observables. So, the scaling laws of the aforementioned kind are relevant for stellar CME studies, that lack spatially resolved observations. In this context, the Güedel-Benz relationship (GBR; Gudel et al. 1993) that connects L_X and microwave (5 - 8 GHz) luminosity by a power-law is noteworthy. GBR revealed the link between the population of flare-accelerated electrons propagating towards the sun causing the microwave and SXR flare, and it provides a common framework to explore the solar and stellar flares (Guedel & Benz 1993; Guedel et al. 1995; Airapetian & Holman 1998). This work will explore analogous correlations between L_X , L_R , and the physical properties of the ‘flare- CME- type-II’ events.

Section 2 presents the data sources and the event catalog. The analysis results and their interpretation are described in Sect. 3, followed by conclusions in Sect. 5.

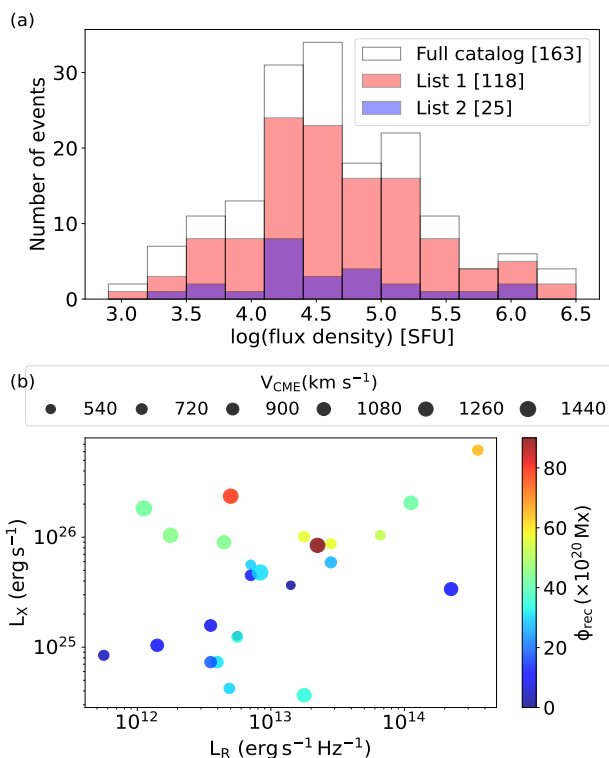


Fig. 1. (a) L_R histogram for DH type-II event lists and the full catalog. Sample sizes are given in square brackets. The distributions look similar across event lists with a median flux density of $10^{4.5}$ SFU. (b) Properties of List 2 events. L_X is not well correlated with L_R .

2. Data and Methodology

We use the calibrated decameter-hectrometric (DH) radio dynamic spectra from Radio and Plasma Wave Investigation (WAVES) instruments onboard Wind, STEREO A and STEREO B spacecraft. The DH type-IIs were chosen for this study over the metric bursts for multiple reasons. Firstly, the DH type-II bursts are produced at regions of CME-driven interplanetary shocks caused by relatively stronger flares and faster CMEs than their metric counterparts (Gopalswamy 2011; Miteva et al. 2017). Also, DH bursts are more closely associated with SEPs and sustained gamma ray emission than the type-IIs confined within the metric band, making these bursts more interesting from a space weather perspective (Gopalswamy 2006; Miteva et al. 2017; Gopalswamy et al. 2018b). The database of the calibrated DH dynamic spectra from each spacecraft gathered over multiple solar cycles forms a uniformly calibrated long-term dataset. Besides, simultaneous STEREO and Wind observations form a unique multi-vantage point radio database.

The events reported in the DH type-II catalog (Gopalswamy et al. 2019) between Nov 2006 and July 2023 form our initial sample. For each event, the data from all spacecraft were examined and the best-recorded event dynamic spectrum (DS) was chosen for further analysis. The peak flux of the burst within 3 - 7 MHz range was used to compute L_R . Within 3 - 7 MHz the major burst types include type-II and type-III bursts. The long-duration broadband type-IV bursts are mostly confined above 7 MHz (Mohan et al. 2024a). Type-IIIs have very high frequency-drift rates than type-IIs making it easier to disentangle the emission from near-simultaneous bursts in the DS. So, 3 - 7 MHz range is an ideal choice to ensure robust flux estimation in most cases with minimal impact from co-temporal bursts. L_X and CME parameters were obtained by cross-matching the radio event list with the publicly available catalogs maintained

by the Coordinated Data Analysis Workshops (CDAW) group¹. The resultant catalog (hereafter ‘full catalog’) has 163 bursts with 3- 7 MHz band L_R estimates. Of these, 118 events (List 1) have good estimation of L_X and CME properties, especially its speed (V_{CME}). From List 1, the type-II bursts that are well isolated in the DS, without potential contamination from other emission features were chosen. ϕ_{rec} was estimated for all possible events using the post eruption arcade (PEA) method described in Gopalswamy et al. (2017). All these resulted in a list of 25 ‘flare- CME- type-II’ events (List 2) with reliable estimates for L_X, L_R, V_{CME} , and ϕ_{rec} , which will be the focus of our analysis. The full catalog, event lists, the calibrated DS across the entire WAVES band and within 3 - 7 MHz, and a detailed description of the tables are made available online².

3. Results

Figure 1a shows the histogram of L_R for the full catalog and the sub-lists. The median flux density is $\sim 3.5 \times 10^4$ SFU for all lists. Figure 1b shows the flare and CME properties of List 2 events. There is no significant correlation between L_R and L_X . However, certain other parameter pairs appear to be correlated, which will be discussed in detail. Two novel metrics of ‘flare power’ ($P_{flare} = \sqrt{L_X \phi_{rec}}$) and ‘CME power’ ($P_{CME} = \sqrt{L_R V_{CME}^2}$) are defined by combining ϕ_{rec} and V_{CME} , with L_X and L_R respectively. Spearman cross-correlation analysis was carried out between each of the parameters considered, namely $L_X, L_R, \phi_{rec}, V_{CME}, P_{flare}$ and P_{CME} . Given the small sample size, a bootstrap technique was employed to obtain robust correlation coefficients and p-values. Of the 25 events, 18 ($\sim 70\%$) were randomly selected iteratively while computing cross-correlations, $C_{R,C}$, between each parameter pair (R,C) over 10000 rounds. The median of the 10000 $C_{R,C}$ and p vales for each pair was noted. The errors on each median $C_{R,C}$ is the median absolute deviation. Fig. 2a shows the cross-correlation matrix with each cell providing the median $C_{R,C}$ between the respective row (R) and column (C) parameters. The cell color denote the median p-value, and the text color of $C_{R,C}$ denotes the statistical significance. $|C_{R,C}|$ values with $p < 0.015$ are considered very significant (green text), while $0.015 < p < 0.05$ are significant (white text) and those with $p > 0.05$ are insignificant (red text). The $C_{R,C}$ for the tuples (P_{flare}, L_X), (P_{flare}, ϕ_{rec}), (P_{CME}, L_R) and (P_{CME}, V_{CME}) are ignored (marked by ‘-’) since the row parameter is a function of the column parameter by definition.

The parameter tuples with significant correlation include (ϕ_{rec}, L_X), (P_{CME}, P_{flare}), (P_{CME}, L_X) and (P_{CME}, ϕ_{rec}). Our $C_{L_X, \phi_{rec}}$ matches the recent result by Kazachenko (2023). However, both L_X and ϕ_{rec} are related to the low-coronal flare, while we aim to identify links between the powering of the low-coronal flare and the interplanetary type-II burst. This makes the other significantly correlated tuples more interesting.

3.1. Flare - CME - type-II scaling laws

Of all possible parameter pairs where one is purely related to the SXR flare and the other to radio burst physics, the engineered novel power metrics (P_{flare}, P_{CME}) shows the highest correlation. We derive a new robust scaling law between the powers in the

¹ <https://cdaw.gsfc.nasa.gov/>

² https://cdaw.gsfc.nasa.gov/CME_list/radio/multimission_type2/

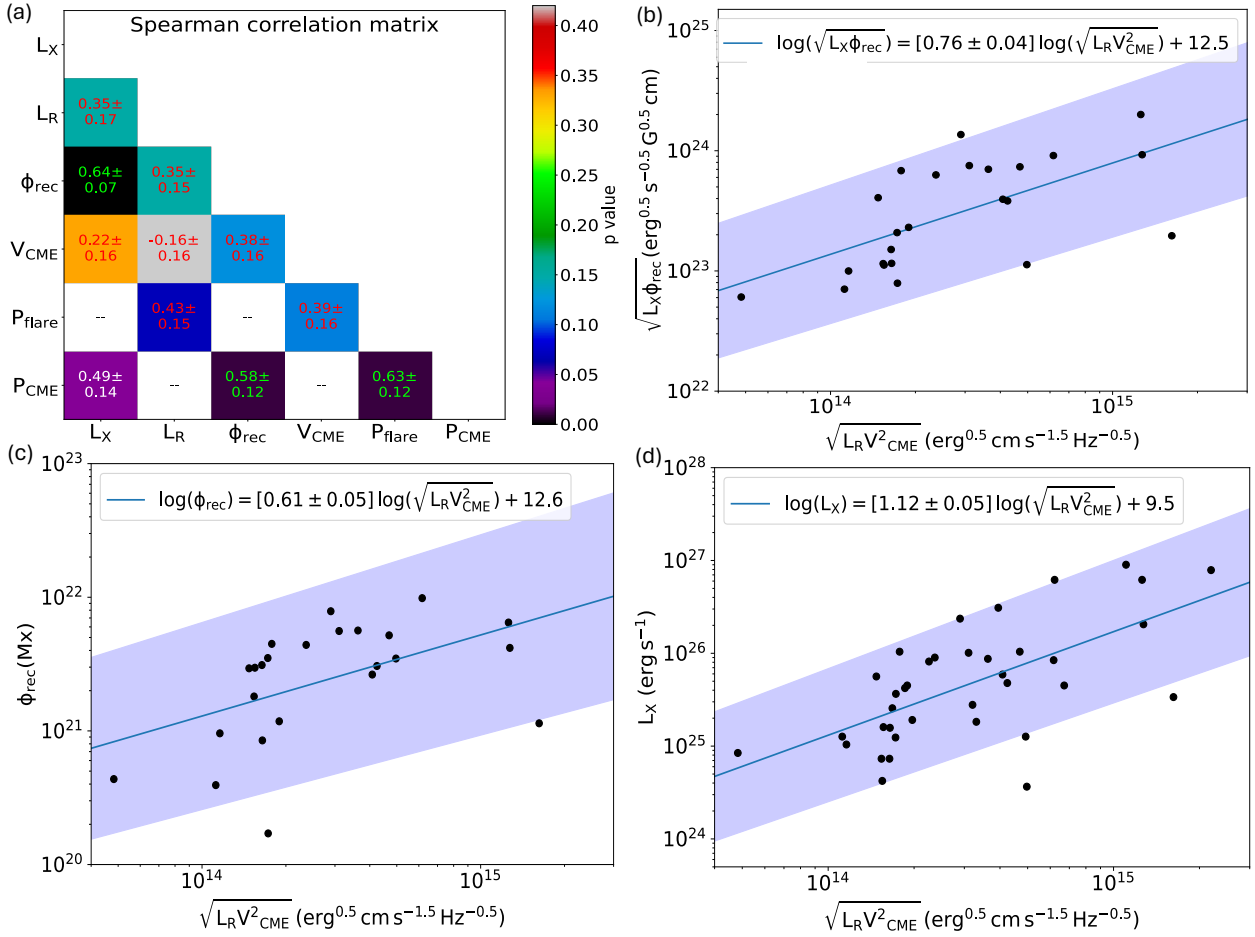


Fig. 2. (a) Spearman Correlation matrix. Each cell provides the Spearman cross-correlation coefficient ($C_{R,C}$) between the respective row (R) and column (C) parameter. Cell color represents the p-value. The $C_{R,C}$, error and p-value are found using a Bootstrapping analysis. The ‘very significant’ ($p < 0.015$), ‘significant’ ($p < 0.05$) and ‘insignificant’ ($p > 0.05$) $C_{R,C}$ values are shown in green, white and red texts respectively. The $C_{R,C}$ between the novel composite metrics and the respective dependent parameters are masked and marked by ‘-’. (b) $P_{\text{flare}} - P_{\text{CME}}$ Scaling law. (c-d) Relations connecting ϕ_{rec} and V_{CME} to P_{CME} . Shaded regions represent the fitting uncertainty intervals.

flare- and CME-driven activity,

$$\sqrt{L_X \phi_{\text{rec}}} = (1 \pm 8.5) \times 10^{12.5} \left(\sqrt{L_R V_{\text{CME}}^2} \right)^{0.76 \pm 0.04}. \quad (1)$$

Interestingly, the power-law index coincides with that of GBR ($L_X \propto L_R^{0.73}$; Guedel et al. 1995) within estimation errors. GBR computes L_R in the 5 - 8 GHz range. The other significant strongly correlated tuples also provide useful scaling laws;

$$L_X = (1 \pm 10) \times 10^{9.5} \left(\sqrt{L_R V_{\text{CME}}^2} \right)^{1.12 \pm 0.05} \quad (2)$$

$$\phi_{\text{rec}} = (1 \pm 10.4) \times 10^{12.6} \left(\sqrt{L_R V_{\text{CME}}^2} \right)^{0.61 \pm 0.05}. \quad (3)$$

Note that V_{CME} and L_R that make up P_{CME} , have insignificant or no correlation with L_X and ϕ_{rec} , making the strong $P_{\text{CME}} - P_{\text{flare}}$ correlation robust. The power-law indices are constrained fairly well. More data from the ongoing solar cycle will help better constrain both the fit parameters in the future.

4. Discussion

4.1. $P_{\text{CME}} - P_{\text{flare}}$ scaling law: physical insight

The ‘flare-CME-type-II’ phenomenon has two major sub-events, the ‘reconnection-heating’ event in the low corona and the ‘CME

- particle acceleration’ event extending the impact of the energetic phenomenon to higher corona and IP space leading to various space weather effects. The populations of electrons that are accelerated and the powering mechanism are different in both sub-events, though the evolution of the sub-events are causally and physically connected. So, robust scaling relations that connect the various observable features of the sub-events of a ‘flare- CME- type-II’ phenomenon remained elusive (see, Sec. 1). However, our physics-informed ‘feature engineering’ (Hastie et al. 2009) provide novel metrics, P_{CME} and P_{flare} , with a significant correlation between them and with other parameters (Eqns. 1– 3). P_{CME} and P_{flare} are defined as proxies for the mean power in each sub-event using direct observables: P_{flare} for the ‘reconnection-heating’ event and P_{CME} for the ‘CME-particle acceleration’ event. The reconnected magnetic flux (ϕ_{rec}) leads to strong local electric fields accelerating electrons in the coronal post-flare loops leading to local heating and associated thermal SXR flares. So L_X is a direct proxy to the heating power. Meanwhile, ϕ_{rec} is a direct observable proxy to the power in the reconnection-driven electric fields. P_{flare} estimates the coupled power in the reconnection and heating, providing a proxy to the power in the ‘reconnection-heating’ event. Meanwhile, the erupted CME following the reconnection event generates a propagating shock, accelerating electrons and causing the type-II burst. The direct observable proxy to the CME shock power is the V_{CME}^2 and L_R is the proxy to the particle

celeration strength. Like P_{flare} , P_{CME} estimates the typical power of the ‘CME-particle acceleration’ event. The novel power metrics, P_{flare} and P_{CME} , thus couple the strength of local particle acceleration with the power in their respective drivers, for the two major sub-events of a ‘flare-CME-type-II’ phenomenon.

4.2. Comparison with GBR

The $P_{\text{CME}}-P_{\text{flare}}$ scaling law can be philosophically compared to the GBR. GBR relates the power in the low coronal particle acceleration ($L_{R,5-8\text{GHz}}$) and subsequent heating (L_X). Any attempted extension or a search for a similar L_R-L_X scaling has not been quite successful in the meterwave band (e.g. [Callingham et al. 2021](#); [Yiu et al. 2024](#)). A major physical reason for this is that the metric bursts have a variety of dynamic spectral types, with each type having a unique driving mechanism. Here we choose the type-II bursts since they are highly sought after in solar and stellar CME studies. Based on the power in the different drivers of the radio and SXR emission in a ‘flare-CME-type-II’ phenomena, we modify the L_R term in GBR to $\sqrt{L_R V_{\text{CME}}^2}$ and the L_X to $\sqrt{L_X \phi_{\text{rec}}}$. Interestingly, the new power terms show a power law index similar to the GBR, while also accounting for the differing SXR and radio emission formation scenarios ([Guedel et al. 1995](#)). A similar careful analysis for other radio burst types may also provide meaningful correlations involving L_R and L_X .

4.3. Implications to stellar CME studies

Being disk-averaged, L_X and L_R are observable in other stars. Combining these observables and the scaling laws associated with the strong correlations (Eqn. 1, and Eqn. 3 or the $L_X-\phi_{\text{rec}}$ scaling law estimated over a larger sample ([Pevtsov et al. 2003](#))), the unknown V_{CME} and ϕ_{rec} can be estimated. V_{CME} and ϕ_{rec} cannot be directly inferred in stars since flare imaging observations are impossible. Besides, since the type-II emission forms primarily via a plasma emission mechanism, the observation frequency (ν) relates to the local electron density (n_e ; $\nu \propto \sqrt{n_e}$) ([Ginzburg & Zhelezniakov 1958](#); [Melrose 1970](#)), the type-II frequency drift-rate ($\delta\nu/\delta t$) and the V_{CME} estimate can provide the density gradient ($\nabla_h n_e$) across the coronal height (h).

$$\frac{\delta\nu}{\delta t} = \frac{\delta\nu}{\delta n_e} \frac{\delta n_e}{\delta h} \frac{\delta h}{\delta t} \quad (4)$$

$$\frac{\delta\nu}{\delta t} = \frac{\nu}{2n_e} \nabla_h n_e V_{\text{CME}}, \quad (5)$$

where $V_{\text{CME}} = \delta h/\delta t$. V_{CME} , ϕ_{rec} and $\nabla_h n_e$ are crucial constraints to flare and CME evolution models.

Since DH type-IIs and metric bursts have similar emission mechanism, the results presented provide a direction to extend the L_R-L_X correlation studies to the metric band by carefully considering the physical drivers of SXR and radio emission. The extension of the analysis presented here to metric waveband is beyond the scope of this work given the lack of uniformly calibrated long term metric dynamic spectral database, unlike the DH band. However, the results presented provide a scientific motivation to calibrate and explore the metric type-II events using a similar methodology. But, one may have to use the CME speed in the lower coronal heights probed by the meterwave band as opposed to the IP space V_{CME} used here.

Given the low occurrence rate of CME-associated metric bursts in active stars ([Villadsen & Hallinan 2019](#); [Zic et al. 2020](#); [Mohan et al. 2024b](#)), and the expected higher occurrence rate in the DH band ([Alvarado-Gómez et al. 2022](#)), there has been

a push for space-based DH band observatories. Our results aid this initiative by providing the flux estimates of the ‘flare-CME-type-II’ events with varying $V_{\text{CME}}-\phi_{\text{rec}}$ values.

5. Conclusion

We present a novel scaling law for ‘flare-CME-type-II’ events using decades-long solar decameter-hectometric type-II burst data and simultaneous multi-waveband imaging and non-imaging database. The scaling relation links two novel metrics of ‘flare power’ ($P_{\text{flare}} = \sqrt{L_X \phi_{\text{rec}}}$) and ‘CME power’ ($P_{\text{CME}} = \sqrt{L_R V_{\text{CME}}^2}$), where L_X is the SXR luminosity, L_R the type-II luminosity, ϕ_{rec} is the reconnection flux and V_{CME} is the CME speed. When P_{flare} encapsulates the power in low-coronal heating and the associated SXR flare, P_{CME} relates to CME shock-driven particle acceleration and the associated type-II burst. The two terms together encapsulate the two major manifestations of ‘flare-CME-type-II’ event in the low-coronal (‘reconnection-heating’) and interplanetary (‘CME-particle acceleration’) regions. $P_{\text{flare}} \propto P_{\text{CME}}^{0.76}$. Analogous to the Güdel Benz relation (GBR) that links the flare thermal power and the particle acceleration strength driven by the low-coronal reconnection event, the novel law derives a connection between the low coronal and interplanetary (IP) particle acceleration driven by the reconnection and the CME events. Previous attempts to identify GBR-like scaling laws connecting L_X and L_R in metric band using stellar data had not been successful. This study provides a philosophy and approach to extend an L_X-L_R relationship for long-wavelengths by taking into account the emission drivers. Additionally, we derive scaling laws linking L_X and ϕ_{rec} to P_{CME} . By combining the derived and known scaling laws with disk-integrated L_X and L_R observable in stellar events, V_{CME} , ϕ_{rec} can be estimated. Additionally assuming a plasma emission mechanism for type-II bursts, the frequency drift rate can be used to estimate $\nabla_h n_e$ in the corona, once V_{CME} is estimated. The presented study takes a top-down approach to understand the interconnections between the two major sub-events of the ‘flare-CME-type-II’ phenomena. There need to be a bottom-up approach using modeling to understand the emergence of the various scaling laws, particularly the $P_{\text{flare}}-P_{\text{CME}}$ scaling.

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³ https://cdaw.gsfc.nasa.gov/CME_list/

⁴ <https://spdf.gsfc.nasa.gov/>

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